Candidate surname			Other names
Pearson Edexcel nternational Advanced Level	Centre	Number	Candidate Number
Sample Assessment Materials fo	or first te	aching S	eptember 2018
(Time: 1 hour 30 minutes)		Paper R	eference WME02/01
Mathematics International Advance Mechanics M2	ed Sub	sidiar	y/Advanced Level

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A car of mass 900 kg is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{25}$. The car is travelling at a constant speed of 14 m s⁻¹ and the resistance to motion from non-gravitational forces has a constant magnitude of 800 N. The car takes 10 seconds to travel from A to B, where A and B are two points on the road.

(a) Find the work done by the engine of the car as the car travels from A to B. (4)

When the car is at B and travelling at a speed of $14\,\mathrm{m\,s^{-1}}$ the rate of working of the engine of the car is suddenly increased to P kW, resulting in an initial acceleration of the car of $0.7\,\mathrm{m\,s^{-2}}$. The resistance to motion from non-gravitational forces still has a constant magnitude of $800\,\mathrm{N}$.

(b) Find the value of <i>P</i> .	

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		Q
(Total for Question 1 is 8 marks)		 × 1

2. A particle P of mass 0.7 kg is moving in a straight line on a smooth horizontal surface. The particle P collides with a particle Q of mass 1.2 kg which is at rest on the surface. Immediately before the collision the speed of P is $6\,\mathrm{m\,s^{-1}}$. Immediately after the collision both particles are moving in the same direction. The coefficient of restitution between the particles is e.

(a) Show that $e < \frac{7}{12}$ (7)

Given that $e = \frac{1}{4}$

(b) find the magnitude of the impulse exerted on Q in the collision.

(3)

nestion 2 continued	

Question 2 continued	Leav blan	
Question 2 continued		
	Q2	2_
	(Total for Question 2 is 10 marks)	

3. At time t seconds $(t \ge 0)$ a particle P has velocity $\mathbf{vm} \, \mathbf{s}^{-1}$, where

$$\mathbf{v} = (6t^2 + 6t)\mathbf{i} + (3t^2 + 24)\mathbf{j}$$

When t = 0 the particle P is at the origin O. At time T seconds, P is at the point A and $\mathbf{v} = \lambda(\mathbf{i} + \mathbf{j})$, where λ is a constant.

Find

(a) the value of T,

(3)

(b) the acceleration of P as it passes through the point A,

(3)

(c) the distance *OA*.

(5)

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Question 3 continued		
		Q3
	(Total for Question 3 is 11 marks)	

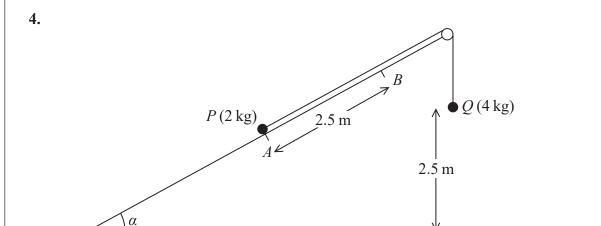


Figure 1

Two particles P and Q, of mass 2 kg and 4 kg respectively, are connected by a light inextensible string. Initially P is held at rest at the point A on a rough fixed plane inclined

at α to the horizontal ground, where $\sin \alpha = \frac{3}{5}$. The string passes over a small smooth

pulley fixed at the top of the plane. The particle Q hangs freely below the pulley and 2.5 m above the ground, as shown in Figure 1. The part of the string from P to the pulley lies along a line of greatest slope of the plane. The system is released from rest with the string taut. At the instant when Q hits the ground, P is at the point B on the plane. The coefficient of friction between P and the plane is $\frac{1}{A}$.

- (a) Find the work done against friction as P moves from A to B. (4)
- (b) Find the total potential energy lost by the system as P moves from A to B. (3)
- (c) Find, using the work-energy principle, the speed of P as it passes through B.

 (4)

estion 4 continued	

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Question 4 continued		
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	Total for Question 4 is 11 marks)	
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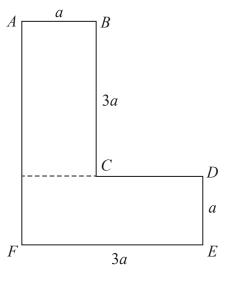


Figure 2

The uniform lamina ABCDEF, shown in Figure 2, consists of two identical rectangles with sides of length a and 3a. The mass of the lamina is M. A particle of mass kM is attached to the lamina at E. The lamina, with the attached particle, is freely suspended from A and hangs in equilibrium with AF at an angle θ to the downward vertical.

Given that $\tan \theta = \frac{4}{7}$, find the value of k.

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Question 5 continued	
	1

Question 5 continued		Leave
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	(Total for Question 5 is 10 marks)	

(5)

6.

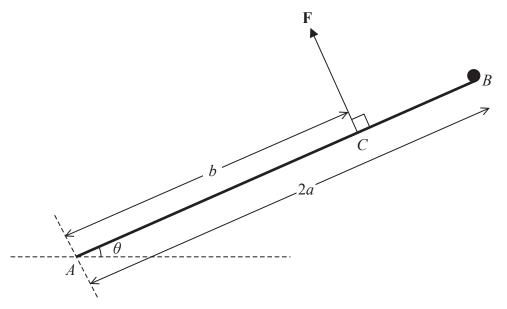


Figure 3

A uniform rod AB, of mass 3m and length 2a, is freely hinged at A to a fixed point on horizontal ground. A particle of mass m is attached to the rod at the end B. The system is held in equilibrium by a force \mathbf{F} acting at the point C, where AC = b. The rod makes an acute angle θ with the ground, as shown in Figure 3. The line of action of \mathbf{F} is perpendicular to the rod and in the same vertical plane as the rod.

(a) Show that the magnitude of **F** is
$$\frac{5mga}{b}\cos\theta$$
 (4)

The force exerted on the rod by the hinge at A is \mathbf{R} , which acts upwards at an angle ϕ above the horizontal, where $\phi > \theta$.

- (b) Find
 - (i) the component of **R** parallel to the rod, in terms of m, g and θ ,
 - (ii) the component of **R** perpendicular to the rod, in terms of a, b, m, g and θ .
- (c) Hence, or otherwise, find the range of possible values of b, giving your answer in terms of a.

estion 6 continued	

Question 6 continued	Leave
	Q6
(Total for Question 6 is 11 marks)	

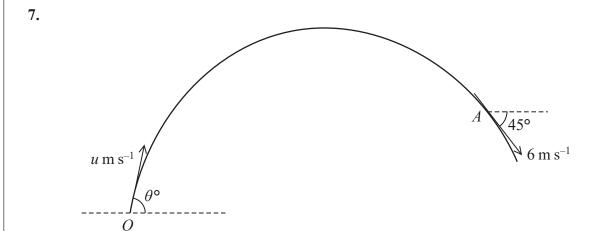


Figure 4

At time t = 0, a particle P of mass 0.7 kg is projected with speed u m s⁻¹ from a fixed point O at an angle θ ° to the horizontal. The particle moves freely under gravity. At time t = 2 seconds, P passes through the point A with speed 6 m s⁻¹ and is moving downwards at 45° to the horizontal, as shown in Figure 4.

Find

(a) the value of θ ,

(6)

(b) the kinetic energy of P as it reaches the highest point of its path.

(3)

For an interval of T seconds, the speed, $v \text{ m s}^{-1}$, of P is such that $v \leq 6$

(c) Find the value of *T*.

(5)

estion 7 continued	

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Question 7 continued	

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Mechanics M2 Mark scheme

Question	Scheme		Marks
1(a)	Resolving parallel to the plane	Condone trig confusion	M1
	$D = 900g\sin\theta + 800$		A1
	$\frac{900}{25}g + 800(=1152.8)$ (N)		
	Work done : Their $D \times$ distance = 1152.8×14×10	Independent. For use of 14 x 10 x their <i>D</i>	M1
	=161392 =161 kJ (160)	Accept 161000 (J), 160000 (J). Ignore incorrect units.	A1
			(4)
	Alternative using energy		
	Work done = $900gd \sin \theta + 800d$	Allow with incorrect d	M1A1
	Use of $d = 14 \times 10$	Independent – allow in an incorrect expression	M1
	$=161392 = 161 \mathrm{kJ} (160)$		A1
			(4)
1(b)	Equation of motion	All terms required. Condone trig confusion and sign errors. Allow with 900 <i>a</i>	M1
	$D - 900g \sin \theta - 800 = 900 \times 0.7$	Correct unsimplified with $a = 0.7$ used Accept with their 1152.8 arising from a 2 term expression in (a)	A1
	$(D-1152.8=900\times0$	0.7)	
	D = 1782.8 (N)		
	Use of $P = Fv$ $P = 14 \times \frac{\text{their } D}{1000}$	Independent Treat missing 1000 as misread, so allow for $14 \times \text{their}D$ Allow for $\frac{1000P}{14}$ (or $\frac{P}{14}$) in their equation of motion	M1
	P = 25.0(25)	cao	A1
	1 - 23.0 (23)	- Cuo	(4)
			8 marks

Question	Scheme		Marks
2(a)	6 m s ⁻¹		
	0.7 kg	1.2 kg	
	CLM: $0.7 \times 6 = 0.7 \times v + 1.2w$	Requires all terms & dimensionally correct	M1
	(42 = 7v + 12w)	Correct unsimplified	A1
	Impact:	Used the right way round Condone sign errors	M1
	w-v=6e		A1
	Equation in e and v only: $42-72e=19v$	Dependent on the two previous M marks	DM1
	Use direction to form an inequality:	Independent. Applied correctly for their <i>v</i>	M1
	$42 - 72e > 0 \Rightarrow e < \frac{7}{12}$	*Given answer*	A1
			(7)
2(b)	Impulse on Q : $I = w \times 1.2$		M1
	Solve for $w: w = v + 6e = \frac{42 - 72 \times \frac{1}{4}}{19} + 6 \times \frac{1}{4}$	Accept unsimplified with e substituted. Have to be using w in part (b) $w = \frac{105}{38} = 2.763$ seen or implied	B1
	$I = 1.2 \times \frac{42}{19} \times \frac{5}{4} = \frac{63}{19} (= 3.32) \text{ (N s)}$	3.3 or better	A 1
			(3)
	Alternative		
	Impulse on $Q = -$ impulse on P	1	
	=-0.7(v-6)	Accept negative here	M1
	$=-0.7\left(\frac{42-\frac{1}{4}\times72}{19}-6\right)$	Substitute for <i>e</i> in their <i>v</i> $v = \frac{24}{19} = 1.263 \text{ seen or}$ implied Accept negative here.	B1

Question	Scheme		Marks
2(b) continued		inal answer must be ositive. .3 or better	A1
			(3)
		(10) marks)

Question	Scheme		Marks
3(a)	Use $\mathbf{v} = \lambda (\mathbf{i} + \mathbf{j})$: $6T^2 + 6T = 3T^2 + 24$	Form an equation in t, T or λ $\lambda^2 - 108\lambda + 2592 = 0$	M1
	Solve for $T 3T^2 + 6T - 24 = 0$,	Simplify to quadratic in t , T or λ and solve.	M1
	(T+4)(T-2)=0, T=2	T = 2 only	A1
	If they score M1 and then state $T = 2$ al	low 3/3	
	If they guess $T = 2$ and show that it wo	rks then allow 3/3.	
	If all we see is $T = 2$ with no equation the available for (b) and (c).	nen 0/3 for (a) but full marks are	
		1	(3)
3(b)	Differentiate: $\mathbf{a} = (12t + 6)\mathbf{i} + 6t\mathbf{j}$	Majority of powers going down Need to be considering both components	M1
		Correct in t or T	
	$=30i+12j \text{ (m s}^{-2})$	Cao	A1
			(3)
3(c)	Integrate: $\mathbf{r} = (2t^3 + 3t^2(+A))\mathbf{i} + (t^3 + 24t(+B))\mathbf{j}$	Clear evidence of integration. Need to be considering both components. Do not need to see the constant(s).	M1
	-1 each error		A2
	If the integration is seen in part (a) it scores no marks at that stage, but if the result is used in part (c) then the M1A2 is available in part (c)		
	$\mathbf{O}\mathbf{A} = 28\mathbf{i} + 56\mathbf{j}$ Use their T		
	Dependent on previous M1 Use of Pythagoras on their OA		DM1
	63 or better, $\sqrt{3920}$		A1
NB: Incorrect T can score 2/3 in (b) and 4/5 in (c)		d 4/5 in (c)	
			(5)
			11 marks)

Question	Scheme		Marks
4(a)	Resolve perpendicular to the plane: $R = 2g \cos \alpha$		B1
	Use $F = \mu R$: $F = \frac{1}{4} \times 2g \times \frac{4}{5} \left(= \frac{2g}{5} \right)$	with $\frac{1}{4}$ and their R (3.92)	M1
	Work done: WD = $2.5 \times F$	For their <i>F</i>	dM1
	$=2.5 \times \frac{2g}{5} = 9.8 (\mathrm{J})$	Accept g	A1
	If a candidate has found the total work done but you can see the correct terms/processes for finding the work done against friction, give B1M1DM1A0 (3/4)		
			(4)
4(b)	Change in PE: $\pm (4g \times 2.5 - 2g \times 2.5 \sin \alpha)$	Requires one gaining and one losing Condone trig confusion	M1
	$=\pm (4g\times 2.5 - 2g\times 1.5)$	± (correct unsimplified)	A1
	PE lost = $7g = 68.6$ (J)	or 69 (J) Accept 7g	A1
			(3)
4(c)	KE gained + WD = loss in GPE	The question requires the use of work-energy. Alternative methods score 0/4. Requires all terms but condone sign errors (must be considering both particles)	M1
	$\frac{1}{2} \times 4v^2 + \frac{1}{2} \times 2v^2 + (\text{their (a)}) = (\text{their (b)})$	Correct unsimplified1 each error	A2
	$3v^2 = 6g$		
	$v = \sqrt{2g} = 4.43 (\text{m s}^{-1})$	or 4.4. Accept $\sqrt{2g}$	A1
	·	<u>'</u>	(4)
	Alternative		
	Equations of motion for each particle leading 12σ	particle leading to	
	to $T = \frac{12g}{5} = 23.52$ followed by a W-E	$T = \frac{12g}{5} = 23.52$ followed	
	equation for <i>P</i> :	by a W-E equation for <i>Q</i> :	
	$2.5T = \frac{1}{2} \times 2v^2 + 2g \times 2.5 \sin \alpha + (a) \text{ M1A2}$	$\frac{1}{2} \times 4v^2 + 2.5T = 4g \times 2.5$	
	$v = \sqrt{2g} = 4.43 (\text{m s}^{-1})$		A1

Question	Scheme	Marks
4(c) continued	Use of $\alpha = 36.9$ gives correct answers to 3 sf	
	Use of $\alpha = 37$ gives correct answers to 2 sf and more than this is not justified, so A0 if they give 3 sf in this case.	
	(1	1 marks)

Question	Scheme		Marks
5	Moments about vertical axis (AF):	Requires all terms and dimensionally correct but condone <i>g</i> missing	M1
	$\frac{Mg}{2} \times \frac{1}{2} a + \frac{Mg}{2} \times 1.5a + 3akMg = Mg(1+k)\overline{x}$	-1 each error Accept with <i>M</i> and/or <i>g</i> not seen.	A2
	$\left(\overline{x} = \frac{1+3k}{1+k}a\right)$		
	Moments about horizontal axis (AB or FE):	Requires all terms and dimensionally correct but condone <i>g</i> missing	M1
	$\frac{Mg}{2} \times 1.5a + \frac{Mg}{2} \times 3.5a + 4akMg = Mg(1+k)\overline{y}$	-1 each error. Accept with <i>M</i> and/or <i>g</i> not seen. Do not penalise repeated errors.	A2
	$\left(\overline{y} = \frac{2.5 + 4k}{1 + k}a\right)$		
		Working with axes through F gives $\overline{x} = \frac{1+3k}{1+k}a \text{ and }$	
		$\overline{y} = \frac{1.5}{1+k} a$	
	SR: A candidate working with a mixture of mass and mass ratio can score 4/6 M1A0A0M1A2		
	Use of $\tan \theta$ with their distances from AF & AB	Must be considering the whole system. Allow for inverted ratio.	M1
	$\tan \theta = \frac{M + 3kM}{2.5M + 4kM} \left(= \frac{4}{7} \right)$	or exact equivalent	A1
	Equate their $\tan \theta$ to $\frac{4}{7}$ and solve for k :		M1
	$7M + 21kM = 10M + 16kM$ $k = \frac{3}{5}$	cso	A1
	5	C30	(10)
	Alternative for the people who start by consider	ering only the Lishane	(10)

Question	Scheme		Marks
5 continued	$\overline{x} = a$ and $\overline{y} = \frac{5}{2}a$ or $\frac{3}{2}a$	M1 (for either) requires all terms and dimensionally correct but condone g/M missing. A1 for each correct.	M1A2
	Combine with the particle	M1 (for both) requires all terms and dimensionally correct but condone <i>g</i> missing. A1 for each correct.	M1A2
	See over for a more geometrical approach		
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Candidate starts by finding centre of mass at $\left(a, \frac{3}{2}a\right)$ relative to F (or equivalent), M1A2 scored	
	θ	Use of $\tan \theta$ with their distances for finding d_1 or d_2 .	M1 A1
	$\frac{5}{2}a$ a	Obtain length of a side in a triangle containing d_1 $\left(\frac{5}{2}a\right)\tan\theta - a\left(=\frac{3}{7}a\right)$ Correct for their centre of mass	AI

Question	Scheme		Marks	
5 continued		$d_1 = \left(\frac{3}{7}a\right)\cos\theta$ Correct for their centre of mass	A1	
	θ	Use of $\tan \theta$ to find second distance $3a - 4a \tan \theta = \frac{5}{7}a$	M1	
	3a	$d_2 = \frac{5}{7}a\cos\theta$	A1	
	Moments about A: $Md_1 = kMd_2$		M1	
	$\frac{3}{7}a\cos\theta = k \times \frac{5}{7}a\cos\theta \implies k = \frac{3}{5}$		A1	
		•	(10)	
		(1	0 marks)	

Question	Scheme			Marks
6(a)	Taking moments about A:	Requires all terms - condone trig confusion and sign errors		M1
	$bF = 3mga\cos\theta + mg \times 2a\cos\theta$	-1 each error		A2
	$bF = 5mga\cos\theta$	*Given answer*		A1
	$F = \frac{5mga}{b}\cos\theta$			711
		ı		(4)
6(b)	Component of R parallel to <i>AB</i> : $(R\cos(\phi-\theta))$	Requires all terms - condone trig confusion		M1
	$=3mg\sin\theta+mg\sin\theta=4mg\sin\theta$	Correc	et unsimplified	A1
	Component of R perpendicular to <i>AB</i> :	Requires all terms - condone consistent trig confusion and sign errors		M1
	$(R\sin(\phi-\theta)) + F = 4mg\cos\theta$	Correct unsimplified		A1
	Alternatives for: $M(B)$	$2aR\sin(\phi-\theta) + 3mga\cos\theta = F(2a-b)$		M1A1
	M(C)	$bR\sin(\phi-\theta) + (2a-b)mg\cos\theta$ $= 3mg(b-a)\cos\theta$		
	$(R\sin(\phi-\theta)) = 4mg\cos\theta - \frac{5mga}{b}$	$\cos \theta$ Correct with F substituted.		A1
	ISW for incorrect work after cor-	rect		(5)
	Alternative			
	$X = F\sin\theta = \frac{5mga}{b}\cos\theta\sin\theta$	Allow with <i>F</i> . Requires all terms - condone trig confusion		M1
	F substituted		A1	
	$Y = 4mg - F\cos\theta = 4mg - \frac{5mga}{b}\cos^2\theta$		Allow with F . Requires all terms - condone trig confusion and sign errors.	
	Correct unsimplified		A1	
	Correct substituted		A1	
			(5)	
6(c)	Use of $R\sin(\phi-\theta) > 0$			M1
	Solve for b in terms of a: $4 > \frac{5a}{b}$, $(2a \ge)b > \frac{5}{4}a$		2a not required CSO	
	T		(2)	
	Special case:			
	Misread of directions in (b)		NB: This MR can score full marks	(2)

Question	Scheme		Marks
6(c)			
continued	For $\varphi > \theta$, $\tan \phi > \tan \theta$		
	$\tan \varphi = \frac{Y}{X} = \frac{4 - \frac{5a}{b}\cos^2\theta}{\frac{5a}{b}\cos\theta\sin\theta} > \tan\theta$		M1
	$4 - \frac{5a}{b}\cos^2\theta > \frac{5a}{b}\sin^2\theta$		
	$4 > \frac{5a}{b} \left(\cos^2 \theta + \sin^2 \theta\right) \implies b > \frac{5}{4}a$	cso	A1
			(2)
(1			1 marks)

Question	Scheme		
7(a)	Equate horizontal components of speeds:		
	$u\cos\theta^{\circ} = 6\cos 45^{\circ} \left(=3\sqrt{2}\right) (4.24)$	Correct unsimplified	A1
	Use suvat for vertical speeds: $u \sin \theta^{\circ} - 2g = -6 \sin 45^{\circ}$	Condone sign errors	M1
	$\left(u\sin\theta = 2g - 3\sqrt{2}\right)$	Correct unsimplified	A1
	Divide to find $\tan \theta$: $\tan \theta = \frac{2g - 6\sin 45}{6\cos 45}$	Dependent on previous 2 Ms. Follow their components.	DM1
	$\left(=\frac{2g-3\sqrt{2}}{3\sqrt{2}}=3.61\right) \Rightarrow$ $\theta = 74.6 (75)$	(u = 15.93)	A1
	0 - 74.0 (73)		(6)
7(b)	At max height, speed = $u \cos \theta$ (= $3\sqrt{2}$ (m s ⁻¹))		B1
	$KE = \frac{1}{2} \times 0.7 \times \left(3\sqrt{2}\right)^2 (J)$	Correct for their v at the top, $v \neq 0$	M1
	= 6.3 (J)	accept awrt 6.30. CSO	A1
			(3)
7(c)	When P is moving upwards at 6 m s ⁻¹	Use suvat to find first time $v = 6$	M1
	$u\sin\theta - gt = 3\sqrt{2}$		A1
	$2g - 3\sqrt{2} - gt = 3\sqrt{2}$	Solve for <i>t</i>	M1
	$t = \frac{2g - 6\sqrt{2}}{g} = 1.13$	Sensitive to premature approximation. Allow 1.14.	A1
	T = 2 - 1.13 = 0.87	CAO accept awrt 0.87	A1
			(5)
	Alternative		
	$6\sin 45 = 0 + gt$	find time from top to A:	M1A1
	$12\sqrt{2}$	Correct strategy	M1
	$T = 2t = \frac{2}{2} = 0.87$	Correct unsimplified	A1
	g		A1
			(5)

Question	Scheme		Marks	
7(c)	Alternative			
continued	$: u \sin \theta = gt \text{ (their } u, \theta \text{)}$	Time	to top	M1
	<i>t</i> = 1.567		A1	
	T = 2(2-1.567)		M1A1	
	= 0.87		A1	
			(5)	
	Alternative			
	Vertical speed at $A = -$ (vertical speed at			M1
	$B) = -\sqrt{36 - \left(3\sqrt{2}\right)^2} = 3\sqrt{2}$	Or use	e the 45° angle	A1
	Use $v = u + at$ for $A \rightarrow B$	Corre	ct use for their values	M1
	$-3\sqrt{2} = 3\sqrt{2} - gT$		A1	
	T = 0.87		A1	
	See below for alt 7d		(5)	
	Alternative 7d			
	$v^2 = \left(3\sqrt{2}\right)^2 + \left(u\sin\theta - gt\right)^2 \le 36$		Form expression for v^2 . Inequality not needed at this stage	M1
			Correct inequality for v^2 .	A1
	$-\sqrt{18} \le u \sin \theta - gt \le \sqrt{18}$			M1
	$\frac{u\sin\theta - \sqrt{18}}{g} \le t \le \frac{u\sin\theta + \sqrt{18}}{g}$			A1
	$T = \frac{u\sin\theta + \sqrt{18}}{g} - \frac{u\sin\theta - \sqrt{18}}{g} = \frac{2\sqrt{18}}{g} = \frac{2\sqrt{18}}{g}$	0.866		A1
				(5)
	(14			4 marks)